

Sample Size for a Comparative Study

When Comparing Two Independent Group Means

To calculate the optimal sample size the following 4 quantities must be pre-specified:

1. Assuming there is a true underlying difference, how certain do you want to be of detecting this? i.e. **Power**, generally we want power = 90% (should be at least 80%)
2. What **significance level** is difference criterion? The cut off below which we will reject the null hypothesis, generally $p=0.05$ (5%).
3. Clinically important **effect size, d**, you wish to detect in the test i.e. the smallest difference in means that it would be clinically meaningful to detect.
4. **Variability i.e. Standard Deviation (SD)** of the outcome if interest

The above information may be obtained from published studies with similar outcomes and populations or you may need to carry out a pilot study. Then;

$$\text{patients per group} = f(\alpha, \beta) \times \frac{2 \times \text{SD}^2}{(d)^2}$$

Where $f(\alpha, \beta) = 7.85$ or 10.5 for 80% or 90% power respectively with 5% significance. Significance (risk of type I error) is almost always set at 5%.

Example: We wish to conduct a study to compare the mean time taken for patients to recover from two different knee arthroplasty procedures with:

- 90% power
- Significance = 0.05
- The clinically meaningful difference we wish to detect is $d = 2.5$ days
- SD = 5 days

With 90% power and 0.05 significance $f(\alpha, \beta) = 10.5$

$$\begin{aligned} \text{patients per group} &= f(\alpha, \beta) \times \frac{2 \times \text{SD}^2}{(d)^2} \\ &= 10.5 \times \frac{(2 \times 5^2)}{(2.5)^2} \\ &= 84 \end{aligned}$$

→ 168 patients overall need to be recruited for the trial

When Comparing Two Independent Proportions

To calculate the optimal sample size the following 4 quantities must be pre-specified:

1. Assuming there is a true underlying difference, how certain do you want to be of detecting this? i.e. **Power**, generally we want power = 90% (should be at least 80%)
2. What **significance level** is difference criterion? The cut off below which we will reject the null hypothesis, generally $p=0.05$ (5%).
3. The assumed proportion that you wish to detect in group 1, p_1
4. The assumed proportion that you wish to detect in group 2, p_2 [$p_1 - p_2$ is the smallest difference in proportions that is clinically important]

Then,

$$\text{patients per group} = \frac{p_1(1-p_1) + p_2(1-p_2)}{(p_2 - p_1)^2} \times f(\alpha, \beta)$$

Example: The UK PACE study compared fitting a single chamber vs. a dual chamber pacemaker on mortality for patients with atrioventricular block. Assumptions:

$p_1 = 0.24$ [24% mortality in single chamber group]

$p_2 = 0.18$ [18% mortality in dual chamber group]

90% power, 5% significance [$f(\alpha, \beta) = 10.5$]

$$\text{patients per group} = \frac{0.35(1-0.35) + 0.15(1-0.15)}{(0.35 - 0.15)^2} 7.85 = 70$$